

Objectives of the Course

The aim of this course is to provide students with the fundamental concepts of fractional calculus and fractional differential equations, to introduce the Riemann–Liouville, Caputo, and Grünwald–Letnikov definitions, and to develop an understanding of their properties. The course also aims to enable students to use the basic properties of fractional integrals, to solve linear fractional differential equations, to interpret initial and boundary value problems, and to apply Laplace transforms at an introductory level. In this way, students will acquire a solid foundation of fractional differential equations and prepare for applications in engineering and physical sciences.

Course Contents

(1) Introduction to fractional differential equations and their historical development (2) Fundamentals of fractional integrals and derivatives (3) Riemann–Liouville definition and properties (4) Caputo definition and properties (5) Grünwald–Letnikov definition and properties (6) Comparison of different definitions (7) Fractional integral inequalities and fundamental theorems (8) Linear fractional differential equations and sample solutions (9) Initial and boundary value problems (10) Applications of Laplace transforms in fractional differential equations (introductory level) (11) Introduction to basic numerical methods for fractional differential equations (12) Simple examples from physical and engineering applications

Recommended or Required Reading

1 K. S. Miller, B. Ross, An Introduction to the Fractional Calculus and Fractional Differential Equations, John Wiley & Sons, Inc., 1993. 2 I. Podlubny, Fractional Differential Equations, Academic Pres, 1999 3 K. B. Oldham and J. Spanier, The Fractional Calculus, Academic Press, 1974

Planned Learning Activities and Teaching Methods

(1) Theoretical lectures (2) In-class discussions and question–answer activities (3) Problem-solving and sample applications (4) Individual assignments and report writing (5) Presentations and short projects (6) Literature reviews and discussions of recent articles

Recommended Optional Programme Components

(1) Students are advised to attend classes regularly and participate actively. (2) Reviewing knowledge from Analysis I–IV, Linear Algebra, and Differential Equations is beneficial. (3) Following supplementary textbooks, articles, and lecture notes provided during the course is recommended. (4) Solving additional problems and examining examples from the literature are encouraged.

Instructor’s Assistants

There is no teaching assistant teaching the course.

Presentation Of Course

(1) The course is delivered face-to-face in the classroom environment. (2) Students first learn the fundamental concepts theoretically from the instructor. (3) Theoretical lectures are reinforced through problem-solving and in-class discussions. (4) Students’ individual learning is supported by assignments and short projects outside the classroom. (5) Current articles from the literature are reviewed to encourage students’ research orientation.

Dersi Veren Öğretim Elemanları

Assoc. Prof. Dr. Mehmet Şenol

Program Outcomes

- 1. Can write the Riemann-Liouville and Caputo definitions of fractional derivatives.
- 2. Can compare the definitions of Riemann-Liouville, Caputo, and conformable fractional derivatives.
- 3. Can define the concepts of conformable fractional derivatives and integrals and explain their fundamental properties.
- 4. Can utilize conformable fractional power series expansions and Laplace transforms in solution methods.
- 5. Can solve conformable fractional differential equations using the sub-equation method
- 6. Can apply the tanh method in solving conformable fractional differential equations.
- 7. Can obtain exact solutions of conformable fractional differential equations using the exponential function method.
- 8. Can solve fractional differential equations using the auxiliary equation method
- 9. Can follow and interpret current research and literature related to conformable fractional differential equations.

Weekly Contents

Order	PreparationInfo	Laboratory TeachingMethods	Theoretical	Practise
1	For this course, students are expected to have a fundamental background in Analysis I–IV, Linear Algebra, and Differential Equations. Familiarity with functions, differentiation and integration, matrix operations, and solutions of differential equations will facilitate the understanding of concepts related to fractional differential equations.	The course is conducted through theoretical lectures and applied problem-solving methods. Students first learn the fundamental concepts from the instructor and then reinforce their knowledge through examples and in-class discussions. Individual assignments, short projects, and literature reviews support self-learning skills and encourage research orientation.	Introduction to fractional differential equations, historical development, and basic concepts	Introduction to fractional differential equations, historical development, and basic concepts

Order	PreparationInfo	Laboratory TeachingMethods	Theoretical	Practise
2	For this course, students are expected to have a fundamental background in Analysis I–IV, Linear Algebra, and Differential Equations. Familiarity with functions, differentiation and integration, matrix operations, and solutions of differential equations will facilitate the understanding of concepts related to fractional differential equations.	The course is conducted through theoretical lectures and applied problem-solving methods. Students first learn the fundamental concepts from the instructor and then reinforce their knowledge through examples and in-class discussions. Individual assignments, short projects, and literature reviews support self-learning skills and encourage research orientation.	Definitions of fractional integrals and fundamental properties	Definitions of fractional integrals and fundamental properties
3	For this course, students are expected to have a fundamental background in Analysis I–IV, Linear Algebra, and Differential Equations. Familiarity with functions, differentiation and integration, matrix operations, and solutions of differential equations will facilitate the understanding of concepts related to fractional differential equations.	The course is conducted through theoretical lectures and applied problem-solving methods. Students first learn the fundamental concepts from the instructor and then reinforce their knowledge through examples and in-class discussions. Individual assignments, short projects, and literature reviews support self-learning skills and encourage research orientation.	Riemann–Liouville definition of fractional integrals and derivatives	Riemann–Liouville definition of fractional integrals and derivatives
4	For this course, students are expected to have a fundamental background in Analysis I–IV, Linear Algebra, and Differential Equations. Familiarity with functions, differentiation and integration, matrix operations, and solutions of differential equations will facilitate the understanding of concepts related to fractional differential equations.	The course is conducted through theoretical lectures and applied problem-solving methods. Students first learn the fundamental concepts from the instructor and then reinforce their knowledge through examples and in-class discussions. Individual assignments, short projects, and literature reviews support self-learning skills and encourage research orientation.	Properties and examples of the Riemann–Liouville derivative	Properties and examples of the Riemann–Liouville derivative
5	For this course, students are expected to have a fundamental background in Analysis I–IV, Linear Algebra, and Differential Equations. Familiarity with functions, differentiation and integration, matrix operations, and solutions of differential equations will facilitate the understanding of concepts related to fractional differential equations.	The course is conducted through theoretical lectures and applied problem-solving methods. Students first learn the fundamental concepts from the instructor and then reinforce their knowledge through examples and in-class discussions. Individual assignments, short projects, and literature reviews support self-learning skills and encourage research orientation.	Caputo definition of fractional derivatives and properties	Caputo definition of fractional derivatives and properties
6	For this course, students are expected to have a fundamental background in Analysis I–IV, Linear Algebra, and Differential Equations. Familiarity with functions, differentiation and integration, matrix operations, and solutions of differential equations will facilitate the understanding of concepts related to fractional differential equations.	The course is conducted through theoretical lectures and applied problem-solving methods. Students first learn the fundamental concepts from the instructor and then reinforce their knowledge through examples and in-class discussions. Individual assignments, short projects, and literature reviews support self-learning skills and encourage research orientation.	Grünwald–Letnikov definition of fractional derivatives	Grünwald–Letnikov definition of fractional derivatives
7	For this course, students are expected to have a fundamental background in Analysis I–IV, Linear Algebra, and Differential Equations. Familiarity with functions, differentiation and integration, matrix operations, and solutions of differential equations will facilitate the understanding of concepts related to fractional differential equations.	The course is conducted through theoretical lectures and applied problem-solving methods. Students first learn the fundamental concepts from the instructor and then reinforce their knowledge through examples and in-class discussions. Individual assignments, short projects, and literature reviews support self-learning skills and encourage research orientation.	Comparison of different fractional derivative definitions	Comparison of different fractional derivative definitions
8	For this course, students are expected to have a fundamental background in Analysis I–IV, Linear Algebra, and Differential Equations. Familiarity with functions, differentiation and integration, matrix operations, and solutions of differential equations will facilitate the understanding of concepts related to fractional differential equations.	The course is conducted through theoretical lectures and applied problem-solving methods. Students first learn the fundamental concepts from the instructor and then reinforce their knowledge through examples and in-class discussions. Individual assignments, short projects, and literature reviews support self-learning skills and encourage research orientation.	Fractional integral inequalities and fundamental theorems	Fractional integral inequalities and fundamental theorems

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9	For this course, students are expected to have a fundamental background in Analysis I–IV, Linear Algebra, and Differential Equations. Familiarity with functions, differentiation and integration, matrix operations, and solutions of differential equations will facilitate the understanding of concepts related to fractional differential equations.	The course is conducted through theoretical lectures and applied problem-solving methods. Students first learn the fundamental concepts from the instructor and then reinforce their knowledge through examples and in-class discussions. Individual assignments, short projects, and literature reviews support self-learning skills and encourage research orientation.	Linear fractional differential equations: introduction and examples	Linear fractional differential equations: introduction and examples
10	For this course, students are expected to have a fundamental background in Analysis I–IV, Linear Algebra, and Differential Equations. Familiarity with functions, differentiation and integration, matrix operations, and solutions of differential equations will facilitate the understanding of concepts related to fractional differential equations.	The course is conducted through theoretical lectures and applied problem-solving methods. Students first learn the fundamental concepts from the instructor and then reinforce their knowledge through examples and in-class discussions. Individual assignments, short projects, and literature reviews support self-learning skills and encourage research orientation.	Initial and boundary value problems in fractional differential equations	Initial and boundary value problems in fractional differential equations
11	For this course, students are expected to have a fundamental background in Analysis I–IV, Linear Algebra, and Differential Equations. Familiarity with functions, differentiation and integration, matrix operations, and solutions of differential equations will facilitate the understanding of concepts related to fractional differential equations.	The course is conducted through theoretical lectures and applied problem-solving methods. Students first learn the fundamental concepts from the instructor and then reinforce their knowledge through examples and in-class discussions. Individual assignments, short projects, and literature reviews support self-learning skills and encourage research orientation.	Applications of Laplace transforms in fractional differential equations (introductory level)	Applications of Laplace transforms in fractional differential equations (introductory level)
12	For this course, students are expected to have a fundamental background in Analysis I–IV, Linear Algebra, and Differential Equations. Familiarity with functions, differentiation and integration, matrix operations, and solutions of differential equations will facilitate the understanding of concepts related to fractional differential equations.	The course is conducted through theoretical lectures and applied problem-solving methods. Students first learn the fundamental concepts from the instructor and then reinforce their knowledge through examples and in-class discussions. Individual assignments, short projects, and literature reviews support self-learning skills and encourage research orientation.	Introduction to basic numerical methods for fractional differential equations	Introduction to basic numerical methods for fractional differential equations
13	For this course, students are expected to have a fundamental background in Analysis I–IV, Linear Algebra, and Differential Equations. Familiarity with functions, differentiation and integration, matrix operations, and solutions of differential equations will facilitate the understanding of concepts related to fractional differential equations.	The course is conducted through theoretical lectures and applied problem-solving methods. Students first learn the fundamental concepts from the instructor and then reinforce their knowledge through examples and in-class discussions. Individual assignments, short projects, and literature reviews support self-learning skills and encourage research orientation.	Simple examples from physical and engineering applications	Simple examples from physical and engineering applications
14	For this course, students are expected to have a fundamental background in Analysis I–IV, Linear Algebra, and Differential Equations. Familiarity with functions, differentiation and integration, matrix operations, and solutions of differential equations will facilitate the understanding of concepts related to fractional differential equations.	The course is conducted through theoretical lectures and applied problem-solving methods. Students first learn the fundamental concepts from the instructor and then reinforce their knowledge through examples and in-class discussions. Individual assignments, short projects, and literature reviews support self-learning skills and encourage research orientation.	General review and evaluation	General review and evaluation

Order	PreparationInfo	Laboratory TeachingMethods	Theoretical	Practise
15	For this course, students are expected to have a fundamental background in Analysis I–IV, Linear Algebra, and Differential Equations. Familiarity with functions, differentiation and integration, matrix operations, and solutions of differential equations will facilitate the understanding of concepts related to fractional differential equations.	The course is conducted through theoretical lectures and applied problem-solving methods. Students first learn the fundamental concepts from the instructor and then reinforce their knowledge through examples and in-class discussions. Individual assignments, short projects, and literature reviews support self-learning skills and encourage research orientation.	General review and evaluation	General review and evaluation

#### Workload

Activities	Number	PLEASE SELECT TWO DISTINCT LANGUAGES
Vize	1	1,00
Derse Katılım	14	4,00
Ödev	4	3,00
Ders Öncesi Bireysel Çalışma	14	2,00
Ders Sonrası Bireysel Çalışma	14	2,00
Ara Sınav Hazırlık	2	4,00
Final Sınavı Hazırlık	1	1,00
Teorik Ders Anlatım	4	4,00
Problem Çözme	4	4,00

#### Assesments

Activities	Weight (%)
Ara Sınav	40,00
Final	60,00

Matematik Ana Bilim Dalı / MATEMATİK ( YÜKSEK LİSANS - TEZLİ ) X Learning Outcome Relation

	P.O. 1	P.O. 2	P.O. 3	P.O. 4	P.O. 5	P.O. 6	P.O. 7	P.O. 8	P.O. 9	P.O. 10	P.O. 11	P.O. 12	P.O. 13	P.O. 14	P.O. 15	P.O. 16	P.O. 17	P.O. 18
L.O. 1																		
L.O. 2																		
L.O. 3																		
L.O. 4																		
L.O. 5																		
L.O. 6																		
L.O. 7																		
L.O. 8																		
L.O. 9																		

Table :

**P.O. 1 :** Analiz, Uygulamalı matematiğin, Geometri ve Cebirin bazı alt toerileri hakkındaki temel teoremleri yeni problemlere uygulayabilir.

- P.O. 2 :** programcılığı 2
- P.O. 3 :** Matematik, fen bilimleri ve kendi dalları ile ilgili konularda yeterli alt yapıya sahiptir ve bu alanlardaki teorik ve uygulamalı bilgileri matematik problemlerin çözümleri için kullanır.
- P.O. 4 :** Bilimsel, matematiksel düşünme yeteneği kazanabilme ve ilgili alanlarda bu bilgiyi kullanabilme.
- P.O. 5 :** Bilimsel, matematiksel düşünme yeteneği kazanabilme ve ilgili alanlarda bu bilgiyi kullanabilme.
- P.O. 6 :** Temel matematiksel beceriler (problem çözme, akıl yürütme, ilişkilendirme, genelleme) ve bu becerilere dayalı yetenekler edinebilme. (Rasyonel düşünme tekniği kazandırabilme)
- P.O. 7 :** Bilim ve teknolojiadaki gelişmeleri izleme ve kendini sürekli yenileme becerisi kazanabilme.
- P.O. 8 :** Bilgiye erişebilme ve bu amaçla kaynak araştırması yapabilme, veri tabanlarını ve diğer bilgi kaynaklarını kullanabilme becerisine sahip olabilme.
- P.O. 9 :** Çalışma hayatında etik sorumlulukların gereklerini yerine getirebilme.
- P.O. 10 :** Bilim tarihi ve bilimsel bilginin üretimiyle ilgili bilgi edinebilme.
- P.O. 11 :** Eleştirel ve yaratıcı düşünmenin ve problem çözme becerilerinin gelişimi için uygun yöntem ve tekniklerle etkinlikler düzenleyebilme.
- P.O. 12 :** Çalışma hayatı ve sosyal yaşam ile ilgili konularda bireysel ve takım çalışmaları yapabilme.
- P.O. 13 :** Alanı ile ilgili konularda düşüncelerini ve konulara ilişkin çözüm önerilerini yazılı ve sözlü olarak aktarabilme.
- P.O. 14 :** Matematiksel bilgi birikimlerini teknolojide kullanabilme.
- P.O. 15 :** Alanındaki bilgileri izleyebilecek ve meslektaşları ile iletişim kurabilecek düzeyde bir yabancı dili geliştirebilme.
- P.O. 16 :** Gerçek dünya problemlerinde Matematiksel prensipleri uygulayabilme.
- P.O. 17 :** Farklı disiplinlerin yaklaşım ve bilgilerini Matematikte kullanabilme.
- P.O. 18 :** Matematik alanındaki bir problemi, bağımsız olarak kurgulayabilme, çözüm yöntemi geliştirebilme, çözebilme, sonuçları değerlendirebilme ve gerektiğinde uygulayabilme.
- L.O. 1 :** Riemann-Liouville ve Caputo kesirli türev tanımlarını yazabilir
- L.O. 2 :** Riemann-Liouville, Caputo ve uyumlu kesirli türev tanımlarını karşılaştırabilir.
- L.O. 3 :** Uyumlu kesirli türev ve integral kavramlarını tanımlayabilir ve temel özelliklerini açıklayabilir.
- L.O. 4 :** Uyumlu kesirli kuvvet serisi açılımlarını ve Laplace dönüşümlerini çözüm yöntemlerinde kullanabilir.
- L.O. 5 :** Uyumlu kesirli diferansiyel denklemleri sub-equation yöntemiyle çözebilir. .
- L.O. 6 :** Uyumlu kesirli diferansiyel denklemlerin çözümünde tanh yöntemini uygulayabilir.
- L.O. 7 :** Üstel-fonksiyon yöntemiyle uyumlu kesirli diferansiyel denklemlerin tam çözümlerini elde edebilir.
- L.O. 8 :** Yardımcı denklem yöntemini kullanarak kesirli diferansiyel denklemleri çözebilir. .
- L.O. 9 :** Uyumlu kesirli diferansiyel denklemlerle ilgili güncel araştırmaları ve literatürü takip edebilir ve yorumlayabilir.